Requirments

Using the Complex 1-Pole Section (CPS) method described in the article

We have to show that the discrete time variant CPS is BIBO stable.
This happens if and only if, there exist a finite M such that

$$\left(\sum_{i=0}^{\infty} \left| h(i) \right| \le M \right) < \infty$$

Where h(k) is the impulse response of the CPS system. In otherwords

We have to prove that the imuplse response is "absolutly summable"

Assumptions

The output of the CPS is defined as:

$$re(t) = in(t) + R(t) \cdot \left(\cos(w(t)) \cdot re(t-1) - \sin(w(t)) \cdot im(t-1) \right) \qquad \text{Eq 1}$$

$$im(t) = R(t) \cdot \left(\cos(w(t)) \cdot im(t-1) + \sin(w(t)) \cdot re(t-1) \right) \qquad \text{Eq 2}$$

Where re(t) and im(t) are the real and imaginary parts respectivly

R(t) is the time-variant radius (resonance). And w(t) is the time-variant angel (cutoff)

Also $R(t) < 1$ and $R(t) \ge 0$

We also define hre(t) and him(t) as the real and imaginary parts of the impulse response h(t) respectivly

To get the impulse response, we set the input:

$$in(t) = \begin{cases} k & t = 0 \\ 0 & t \ne 0 \end{cases}$$

Therefore, assuming the system has zero initial state

That is re(t)=0 and im(t)=0 for all t<0. Substituting in Eq1 and Eq2

$$hre(0) = k \quad him(0) = 0$$

And for all t>0

$$hre(t) = R(t) \cdot \left(\cos(w(t)) \cdot hre(t-1) - \sin(w(t)) \cdot him(t-1) \right)$$

$$him(t) = R(t) \cdot \left(\cos(w(t)) \cdot him(t-1) + \sin(w(t)) \cdot hre(t-1) \right)$$

Proof

The magnitude of the impulse response is

$$\left| h(t) \right| = \left| hre(t) + j \cdot him(t) \right|$$

$$\left| h(t) \right| = \sqrt{(hre(t))^2 + (him(t))^2} \qquad \text{Eq 3}$$

$$\left| h(t) \right| = \sqrt{\left(R(t) \cdot \left(\cos(w(t)) \cdot hre(t-1) - \sin(w(t)) \cdot him(t-1) \right) \right)^2 + \left(R(t) \cdot \left(\cos(w(t)) \cdot him(t-1) + \sin(w(t)) \cdot hre(t-1) \right) \right)^2}$$

Simplifying

$$|h(t)| = \sqrt{R(t)^2 \cdot \left(hre(t-1)^2 \cdot \left((\cos(w(t)))^2 + \sin(w(t))^2 \right) + him(t-1)^2 \cdot \left((\cos(w(t)))^2 + (\sin(w(t)))^2 \right) \right)}$$

Since $(\cos(w(t)))^2 + (\sin(w(t)))^2 = 1$

$$|h(t)| = R(t) \cdot \sqrt{\left((hre(t-1))^2 + (him(t-1))^2 \right)}$$

Side note: the impulse response magnitude is independant of w(t) (cutoff) changes

Notice, That the square root term on the right is esentially the magnitude of h(t-1). Check Eq3. So we can rewrite it

$$|h(t)| = R(t) \cdot |h(t-1)|$$

So we can expand this back to h(0)

$$|h(t)| = \left(\prod_{i=1}^{t} R(t) \right) \cdot |h(0)|$$

And since $|h(0)| = |k|$

$$|h(t)| = |k| \cdot \prod_{i=1}^{t} R(t)$$

it is easy to see from here that

$$\lim_{t \to \infty} \prod_{i=1}^{t} R(t) = 0 \quad \text{for} \quad 0 \le (R(t) < 1)$$

But for the sake of clarity, we define Rm To be the maximum of $R(t)$ for all t

So $\prod_{i=1}^{t} R(t) \le (Rm)^t$

Therefore

$$\left(|h(t)| = |k| \cdot \prod_{i=1}^{t} R(t) \right) \le |k| \cdot (Rm)^t$$

We know that

$$\lim_{t \to \infty} |k| \cdot (Rm)^t = 0 \quad \text{for} \quad 0 \le (Rm < 1)$$

Therefore

$$\lim_{t \to \infty} |h(t)| = 0$$

Therefore

this esentially means that there exist M such that

$$\left(\sum_{i=0}^{\infty} \left| h\,(i) \right| \le M \right) < \infty$$

Which is what we wanted to prove.